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Graph Theory, SEC-II

Problem Set 1 (RSG)

1) Draw a graph with 4 vertices such that two vertices are of degree 4, one is of degree 5 and another is of degree 3. How many edges are there in this graph?

2) Write the degree sequence of a null graph with 3 vertices. Write the degree sequence of K_n, P_n, C_n where $n \in \mathbb{N}$.

3) Let G be a k-regular graph. If k is odd, then show that G has even no. of vertices.

4) How many vertices are there in a graph with 15 edges if each vertex is of degree 3? Does there exist a graph with 20 edges if each vertex is of degree 3?

5) Does there exist a graph with 4 edges and deg. seqn $(4, 3, 2, 1)$?

6) Show that the number of edges of a 5-regular graph G is a multiple of 5.

7) Find the degree sequence of all possible simple graphs with exactly 3 vertices.

8) Draw a graph (not necessarily simple) with the degree seqn. $(5, 5, 5, 4, 4, 3, 0)$. Draw another graph with deg. seqn $(6, 6, 4, 1, 1)$.

9) Let G be a graph with n vertices and $n-1$ edges. Show that either G has an isolated vertex or a vertex of degree 1.

10) Does there exist simple graphs with following degree sequences? Justify your answer in each case:

    (i) $(5, 4, 2, 1)$
    (ii) $(3, 3, 3, 1)$
    (iii) $(2, 2, 2, 2)$
    (iv) $(4, 3, 2, 1, 0)$
    (v) $(4, 4, 4, 3, 3, 2)$
    (vi) $(4, 4, 3, 2, 2, 1)$
    (vii) $(5, 5, 4, 2, 2, 2)$

11) Draw a graph with given properties or explain why no such graph exists:

    (i) Five vertices each of degree 4
    (ii) Simple graph with 7 edges and 9 vertices with no isolated vertex.
    (iii) Six vertices and six edges with deg. seqn. $(1, 1, 2, 4, 5, 5)$.
    (iv) Seven vertices having degrees $3, 5, 2, 7, 4, 6, 8$.
    (v) Simple graph with 5 vertices such that every vertex is incident on one edge but no two edges are adjacent.
    (vi) Simple graph with 5 vertices and 7 edges.

12) Find the complement of the cycle graph C_5.

13) Give an example of a graph G such that it is same as its complement $\overline{G}$.

14) Does there exist a complete graph with 20 vertices and 180 edges?

2